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From:	General Secretariat of the Council
To:	Permanent Representatives Committee/Council
Subject:	Assessment of Effective Action under the Excessive Deficit Procedure

Delegations will find attached the second part of the note on Improving the Assessment of Effective Action in the Context of the Excessive Deficit Procedure – A Specification of the Methodology.

## Annex 1: Analytical derivation of the refined $\beta$ parameter

### 1) Simplifying the notation

The formula originally used for the beta parameter can be written as follows:

$$\beta_t^{\text{current}} = \frac{(\Delta R_t - DM_t - \eta^R y_t \cdot R_{t-1}) - (\Delta R_t - DM_t - \eta^R y_t \cdot R_{t-1})^{res}}{Y_t^*}$$

For the sake of simplicity and clarity the above formula introduces some minor difference in notation compared to the body of the note. In particular,  $\eta^R$  stands for the elasticity of revenues to the output gap (and replaces  $\varepsilon^*$  previously used, as it is customary to refer to semi-elasticities, e.g. in the computation of the cyclically adjusted balance). All quantities are expressed in nominal terms, unless otherwise noted. Therefore,  $y_t$  represents growth of nominal GDP (and replaces  $y_t^{\text{nom}}$  previously used), and  $Y_t^*$  potential output at current prices.<sup>1</sup> The other symbols and conventions remain unchanged:  $R_t$  - the level of government revenues;  $DM_t$  - the level of discretionary revenue measures; capital letters denote levels and lower case letters growth rates.

With the new notation the formula of the agreed refined version of the  $\beta$  parameter becomes:

$$\beta_t^{\text{new}} = \frac{(\Delta R_t - DM_t - [y_t + (\eta^R - 1)\Delta OG_t] \cdot R_{t-1}) - (\Delta R_t - DM_t - [y_t + (\eta^R - 1)\Delta OG_t] \cdot R_{t-1})^{res}}{Y_t}$$

### 2) Analytical derivation

#### a) Definition of cyclically-adjusted revenues

The cyclically adjusted revenue (CAR) of the commonly agreed methodology is equal to:

$$CAR_t = \frac{R_t}{Y_t} - \varepsilon_R \cdot OG_t$$

where  $\frac{R_t}{Y_t}$ ,  $OG_t$  and  $\varepsilon_R$  respectively are revenues-to-GDP current ratio, current output gap and the revenue semi-elasticity. The output gap and the revenue semi-elasticity are respectively equal to:

$$OG_t = \frac{Y_t - Y_t^*}{Y_t^*} \quad \varepsilon_R = \left( \frac{R_0}{Y_0} \right) \cdot (\eta^R - 1)$$

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<sup>1</sup> It replaces the equivalent expression  $GDP_t^{\text{potential}} \cdot \left( \frac{GDP_t^{\text{nom}}}{GDP_t^{\text{real}}} \right)$  previously used in the denominator.

where  $Y_t$ ,  $Y_t^s$  and  $\eta^R$  respectively are the actual output level, the potential output level and the elasticity of revenues to the output gap. The revenues-to-GDP ratio that is used in the computation of  $\varepsilon_R$  is a 10-year average and is therefore time invariant: its marked by a subscript "0".<sup>2</sup>

b) Decomposition of the change in cyclically-adjusted revenues

The change in cyclically adjusted revenues is equal to:<sup>3</sup>

$$\Delta CAR_t = \Delta \left( \frac{R_t}{Y_t} \right) - \varepsilon_R \cdot \Delta OG_t = \frac{\Delta R_t - y_t \cdot R_{t-1}}{Y_t} - \left( \frac{R_0}{Y_0} \right) \cdot (\eta^R - 1) \cdot \Delta OG_t$$

where  $y_t = \frac{\Delta Y_t}{Y_{t-1}}$  is the nominal output growth, and

$$\Delta \left( \frac{R_t}{Y_t} \right) = \frac{R_t}{Y_t} - \frac{R_{t-1}}{Y_{t-1}} = \frac{R_{t-1} + \Delta R_t}{Y_{t-1} + \Delta Y_t} - \frac{R_{t-1}}{Y_{t-1}} = \frac{Y_{t-1} \cdot \Delta R_t - R_{t-1} \cdot \Delta Y_t}{Y_{t-1} \cdot Y_t} = \frac{\Delta R_t - y_t \cdot R_{t-1}}{Y_t}$$

After adding and subtracting  $\frac{DM_t}{Y_t}$  we can decompose the change in  $CAR$  in order to isolate the discretionary part of the change in revenues-to-output ratio:

$$\Delta CAR_t = \frac{DM_t}{Y_t} + \frac{\Delta R_t - DM_t - y_t \cdot R_{t-1}}{Y_t} - \left( \frac{R_0}{Y_0} \right) \cdot (\eta^R - 1) \cdot \Delta OG_t$$

And after further rearrangements<sup>4</sup>:

$$\Delta CAR_t = \underbrace{\frac{DM_t}{Y_t}}_{(i)} + \underbrace{\frac{\Delta R_t - DM_t - y_t \cdot \eta^R \cdot R_{t-1}}{Y_t}}_{(ii)} + \underbrace{(\eta^R - 1) \cdot \left[ \frac{R_{t-1}}{Y_t} y_t - \left( \frac{R_0}{Y_0} \right) \cdot \Delta OG_t \right]}_{(iii)}$$

where  $DM_t$  are the discretionary revenue measures in year  $t$ .

<sup>2</sup> Further details on the methodology for cyclically-adjusted balances can be found in *The cyclically adjusted budget balance used in the EU fiscal framework: an update* (DG ECFIN Economic Papers 478, March 2013).

<sup>3</sup> It is recalled that all the displayed quantities are nominal, except the output gap which is expressed in real terms.

<sup>4</sup>  $\Delta CAR_t = \frac{DM_t}{Y_t} + \frac{\Delta R_t - DM_t - y_t \cdot R_{t-1}}{Y_t} - \left( \frac{R_0}{Y_0} \right) \cdot (\eta^R - 1) \cdot \Delta OG_t$

$\Delta CAR_t = \frac{DM_t}{Y_t} + \frac{\Delta R_t - DM_t - y_t \cdot R_{t-1}}{Y_t} - (\eta^R - 1) \cdot \frac{R_{t-1}}{Y_t} y_t + (\eta^R - 1) \cdot \frac{R_{t-1}}{Y_t} y_t - \left( \frac{R_0}{Y_0} \right) \cdot (\eta^R - 1) \cdot \Delta OG_t$

$\Delta CAR_t = \frac{DM_t}{Y_t} + \frac{\Delta R_t - DM_t - y_t \cdot \eta^R \cdot R_{t-1}}{Y_t} + (\eta^R - 1) \cdot \left[ \frac{R_{t-1}}{Y_t} y_t - \left( \frac{R_0}{Y_0} \right) \cdot \Delta OG_t \right]$